Identifying classical and quantum Markovian causal models from generalized observation

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Jacobs, Kissinger, and Zanasi [JKZ19] described causal models based on Bayesian networks as certain functors between CDU categories, which, like Markov categories [Fri20], capture probabilistic maps synthetically by giving each object a "copying" map. In that categorical presentation of causal Bayesian networks, a complete common cause is pictured as a random variable being copied and then the outputs being used as inputs to multiple subsequent stochastic maps. The observational data, those generated by the composite process with no intervention, are summarized in a single joint state in a stochastic process category. Intervention on a variable is represented by a "cut" endofunctor severing the variable's connections to its parents and then randomizing the variable, yielding a new joint state on all variables, called an "interventional distribution." The problem of causal identification, to infer from observational data the influences of hypothetical interventions, is posed as the problem of computing from the original state the new state produced by the "cut" endofunctor.

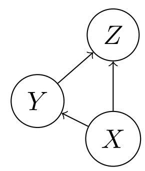
Motivated by recent interest in causal inference with quantum processes, two of the present authors used a compositional framework to generalize classical causal identification techniques for well-known classes of scenarios involving latent confounders to the quantum setting [FK22]. The usual setup for causal identification, including its categorical formulation in [JKZ19], makes two assumptions that fail in the quantum setting. The first is that there exists a meaningful notion of "passive observation," whereby one can start from purely observational data and attempt to predict the results of interventions, and the second is the universal availability of "copying" processes, which enable a single classical random variable to serve as a common cause of several others.

To remove these assumptions, in [FK22], we modified the framework of [JKZ19] by replacing the role played by Bayesian networks (i.e., probabilistic states subject to certain factorisation constraints) with *combs* [CDP08], which are second-order processes taking first-order processes (i.e., local interventions) as inputs. The causal quantity of interest is not an interventional distribution but an *interventional channel*, which maps arbitrary interventions to outcome statistics. The role of "observational data" was played by enumerating the image of this channel under a restricted class of *probing* processes, whose definition depends on the particular setting (e.g., sharp, non-disturbing observations in the classical case or projective measurements in the quantum). It turns out that even in the absence of passive observation, identifying the interventional channel from its image under the probing processes is an interesting, non-trivial problem.

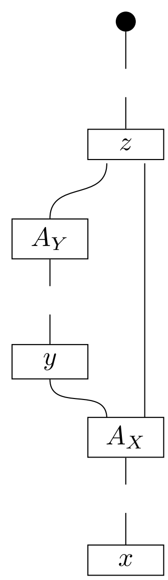
Quantum theory's lack of cloning or broadcasting maps [WZ82; Bar+96] rules out CDU or Markov structure in a quantum process category, making it difficult to apply the techniques of [JKZ19] to a category of quantum causal models. In [FK22] we circumvented this difficulty by eschewing explicit representation of complete common causes and showing that the resulting more generic notion of causal model supported the identification of causal influences for the classes of causal structures we wished to consider. Nevertheless, since statistical causal inference is greatly aided by the so-called "causal Markov assumption," which requires representation of complete common causes, there has remained a question of whether the proposed definition of "quantum complete common cause" in [All+17] is a source of similarly significant inferential power in the quantum setting. Here we answer in the affirmative. Adapting a structure first proposed in the calculus of classical and quantum Bayesian inference introduced by Coecke and Spekkens [CS12], we show in this paper that, although the category **CPM** of quantum processes has no Markov structure, it is possible to work in a larger category of all linear maps, which admits a (non-commutative) Markov-like structure that enables the graphical formulation of quantum common causes.

Using this structure, we will formulate a richer notion of causal model than the one used in [FK22], one that both admits the formulation of complete common causes (hence fully recovering classical causal identification) and allows uniform treatment of the classical and quantum cases. Furthermore, most accounts of classical causal identification, including traditional ones like [Pea09] and the categorical one in [JKZ19], rely, explicitly or not, on the availability of sharp observations of variables in the model, and the fact that the outcome of such an observation can literally be copied to every location in the causal model that depends on it. We show here that by dropping this assumption, one obtains not only a meaningful notion of quantum causal identification, but also new kinds of classical causal identification scenarios wherein one only has access to fuzzy observations, that might give imperfect outcomes and disturb the system in (small) uncontrolled ways.

For any directed acyclic graph G, e.g.,

$G \quad =$ 

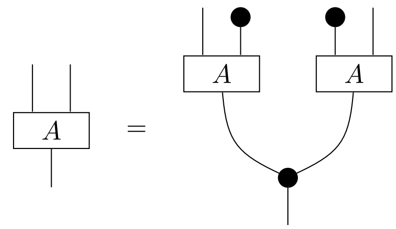
we define a Markovian G-based causal structure as a formal diagram c_G in a free symmetric monoidal category $\mathbf{G}$. In this example,

$c_G \quad =$ 

A G-based causal model then corresponds to a functor F from $\mathbf{G}$ into either a stochastic or a quantum process category, where the formal diagram in $\mathbf{G}$ is mapped to a concrete (classical or quantum) comb $F(c_G)$ that assigns probabilities to outcomes of interventional or observational procedures. The diagram c_G consists of two kinds of maps: those that feed into each location in the comb where an intervention can occur, and those that come out of it. The refinement from previous work is that here, we restrict models F to those sending the maps coming out of each locus of intervention to a distinguished "common cause" subcategory $\mathcal{C}_{cc} \subseteq \mathcal{C}$, whose processes play the role of copying in the classical case for establishing complete common causes. In the classical case, this is the category of functions (considered as a subcategory of all stochastic matrices) and in the quantum case this is the category of partial unitaries (considered as a subcategory of all quantum processes).

In this setting, the causal identification problem is to determine $F(c_G)$, given the graph G and outcome statistics for some restricted probing scheme. Once the model is known, one can predict the results of not only so-called "do" interventions–which discard the state arriving at a locus represented by a vertex in G and prepare in its place a fixed state of the intervener's choice–but also more general kinds of interventions, for which the state leaving a locus depends on the state that has arrived there, and possibly also on events at other loci [SP09; CB20]. Thus the comb conception of causal model expands the class of interventions that can be studied easily.

A key point in the refinement of the notion of causal model from [FK22] is that the processes in the common cause subcategory $\mathcal{C}_{cc}$ in both our classical and quantum examples factor in such a way that each output depends fully on the input, and no "latent confounders." This fact can be formulated classically using the usual copy map, in the style of Markov categories, and in the quantum case using a (not completely positive, non-commutative) broadcasting map like the one that appears in [CS12].



In the classical case, this factorization is immediate from the definition of functions, and for the quantum case, we rely on the characterization of complete quantum common causes in [All+17]. Using this factorization and a calculational trick from [CS12] (namely computing certain "Frobenius inverses" of maps), we give an example of performing causal identification for a non-trivial Markovian model.

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